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SUPERCRITICAL THERMOCAPILLARY CONVECTION REGIMES IN A LIQUID LAYER "ON THE CEILING"

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It has been remarked [1-3] that the equilibrium state of an underside ("on the ceiling") liquid layer heated on its free side can be stable if the force field is sufficiently weak. If it is isothermal in the equilibrium state, the liquid becomes nonisothermal in the presence of perturbations. The resulting thermocapillary forces stabilize long-wavelength perturbations, which are the most hazardous. The equilibrium state has a stability threshold, which determines the critical relation between the destabilizing force of gravity and the stabilizing thermocapillary effect.

In the present article we determine the conditions whereby steady thermocapillary flow generated in a supercritical region forms a planar, single-vortex structure in a layer whose thickness is not constant, but varies monotonically along the layer. We also demonstrate the existence of supercritical regimes with a double-vortex flow structure and a free surface geometry with a convexity or concavity facing downward toward the underlying gaseous phase.

1. LONG-WAVELENGTH EQUATIONS

Let a liquid layer be contained between horizontal solid plates $z = 0$ (ceiling) and $z = d$ (floor) and occupy the region $0 < z < h(x, t)$. Constant temperatures T_0 and T_{10} are specified on the plates $z = 0, d$. The region $h(x, t) < z < d$ is occupied by a heat-conducting gas of negligible density and viscosity. The thermal conductivities λ, λ_1 and thermal diffusivities χ, χ_1 of the liquid and the gas (respectively), the kinematic viscosity ν and density ρ of the liquid, and the acceleration of gravity g are assumed to be constant. The liquid—gas system exists in a low-gravity environment.

The coefficient of surface tension σ is assumed to be a linear function of the temperature T:

$$\sigma = \sigma_0 - \sigma_T[T - \min(T_{10}, - T_0)]. \tag{1.1}$$

In Eq. (1.1) σ_0 and σ_T are specified positive constants. We propose to investigate two-dimensional flows, for which the velocity vector has two components u and w in the directions of the x and z axes, respectively, neither of which is identically zero. We denote by p, T, and T_1 the pressure of the liquid and the temperatures of the liquid and the gas, respectively. We introduce the dimensionless variables

$$\begin{gathered} x' = x/L,\ z' = z/d,\ h' = h/d,\ t' = Ut/L, \\ u' = u/U,\ w' = w/\varepsilon U,\ p' = pd^2/\rho\nu UL, \\ \theta = [T - \min(T_{10}, T_0)]/[T - \min(T, T)]/|T - T|. \end{gathered} \tag{1.2}$$

Here L is a characteristic longitudinal dimension; we assume that $\varepsilon = d/L$ is a small parameter, and U is a characteristic velocity of thermocapillary motion: $U = \varepsilon|T_{10} - T_0|\sigma/\rho\nu$.

The state of the liquid is described by the Navier–Stokes and convective heat-transfer equations. The temperature in the gaseous phase is determined from the heat-conduction equation. The velocity obeys the no-slip condition on the plate $z = 0$. The kinematic condition is satisfied at the interface, along with two dynamical conditions expressing the fact that the normal component of the stress vector is equal to the capillary pressure and that the tangential stress is equal to the stress produced

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by the variability of the surface tension. The system is augmented with the conditions of continuity of the temperatures and heat fluxes at the interface and by the specification of the temperatures on the solid plates. This system of equations is written in the dimensionless variables (1.2):

at $0 < z < h$

$$\begin{gathered} u_{zz} - p_x = \varepsilon^2\mathrm{Re}(u_t + uu_x + wu_z) - \varepsilon^2 u_{xx}, \\ p_z + \gamma = \varepsilon^2 w_{zz} - \varepsilon^4\mathrm{Re}(w_t + uw_x + ww_z) + \varepsilon^4 w_{xx}, \\ u_x + w_z = 0,\ \theta_{zz} = \varepsilon^2\mathrm{RePr}(\theta_t + u\theta_x + w\theta_z) - \varepsilon^2\theta_{xx}; \end{gathered} \tag{1.3}$$

at $h < z < 1$

$$\theta_{1zz} = \varepsilon^2\chi_*^{-1}\mathrm{RePr}\theta_{1t} - \varepsilon^2\theta_{1xx}; \tag{1.4}$$

at $z = h$

$$\begin{gathered} h_t = w - uh_x,\ -(\alpha - \varepsilon^2\theta)(1 + \varepsilon^2 h_x^2)^{-3/2}h_{xx} + \\ 2\varepsilon^2(w_z - h_x u_z) - 2\varepsilon^4(w_x - h_x u_x)(1 + \varepsilon^2 h_x^2)^{-1}h_x = -p, \\ [u_z + \varepsilon^2(w_x - h_x^2 u_z - \varepsilon^2 h_x^2 w_x + 2h_x w_z - 2h_x u_x)] \times \\ (1 + \varepsilon^2 h_x^2)^{-1/2} = -\theta_x,\ \theta = \theta_1, \\ \theta_z - \varepsilon h_x\theta_x = \lambda_*(\theta_{1z} - \varepsilon h_x\theta_{1x}); \end{gathered} \tag{1.5}$$

at $z = 0$

$$u = w = 0,\ \theta = \theta_0; \tag{1.6}$$

at $z = 1$

$$\theta_1 = \theta_{10}. \tag{1.7}$$

The prime is dropped from the dimensionless variables for convenience; $\mathrm{Re} = UL/\nu$ and $\mathrm{Pr} = \nu/\chi$ are the Reynolds and Prandtl numbers, $\lambda_* = \lambda_1/\lambda$, $\chi_* = \chi_1/\chi$, and

$$\alpha = \varepsilon^2\sigma_0/\sigma_T\,|T_{10} - T_0|,\ \gamma = \pm\,\varepsilon^2\rho g L^2/\sigma_T\,|T_{10} - T_0| \tag{1.8}$$

are dimensionless parameters characterizing the ratio of the capillary forces and hydrostatic pressure to the thermocapillary forces. The plus and minus signs in Eq. (1.8) and elsewhere indicate that the positive direction of the z axis is opposite to the force of gravity (layer on the floor) or in the same direction (layer on the ceiling), respectively.

To obtain long-wavelength approximation equations, it is necessary to pass to the limit $\varepsilon \to 0$ in Eqs. (1.3)-(1.7). Assuming that the quantities $\varepsilon^2\mathrm{RePr}$, $\varepsilon^2\mathrm{Re}$, $\varepsilon^2\chi_*^{-1}\mathrm{RePr}$, and $\varepsilon\lambda_*$ tend to zero and that α and γ remain constant in the limit $\varepsilon \to 0$, from Eqs. (1.3)-(1.7) we obtain a boundary-value problem for the long-wavelength approximation, from which explicit expressions for the pressure, velocity, and temperature of the media in terms of the thickness h are found by integration:

$$\begin{gathered} p = \gamma(h - z) - \alpha h_{xx}, \\ u = (z^2/2 - zh)(\gamma h_x - \alpha h_{xxx}) - z(\theta_h)_x, \\ w = \left(\frac{z^2 h}{2} - \frac{z^3}{6}\right)(\gamma h - \alpha h_{xx})_{xx} + \frac{z^2}{2}\,[\gamma h x^2 - \alpha h_x h_{xx} + (\theta_h)_{xx}]; \end{gathered} \tag{1.9}$$

$$\begin{gathered} \theta = \lambda_*\mathrm{sgn}(T_{10} - T_0)z/(1 - h + \lambda_* h) + \frac{1}{2} - \frac{1}{2}\mathrm{sgn}(T_{10} - T_0), \\ \theta_1 = \mathrm{sgn}(T_{10} - T_0)(z - 1)/(1 - h + \lambda_* h) + \frac{1}{2} + \frac{1}{2}\mathrm{sgn}(T_{10} - T_0). \end{gathered} \tag{1.10}$$

In Eqs. (1.9) θ_h is the temperature on the free boundary. An expression for the function θ_h is obtained by substituting $z = h$ into the first (or second) equation (1.10). An equation for the thickness h(x, t) is obtained from the kinematic condition on the basis of expressions (1.9) and has the form

$$h_t = \frac{\partial}{\partial x}\left[\frac{\lambda_* \mathrm{sgn}(T_{10} - T_0) h^2 h_x}{2(1 - h + \lambda_* h)^2} + \frac{h^3}{3}(\gamma h_x - \alpha h_{xxx})\right]. \tag{1.11}$$

Equations (1.9)–(1.11) describe planar thermocapillary flows in the layer $0 < z < h$ in the long-wavelength approximation. The initial data for the function h are not given, since only steady (time-invariant) flows and the stability of the equilibrium state are investigated below.

The expression in the brackets in Eq. (1.11) gives the mass flow of liquid across the layer. In this article we are interested only in steady-state solutions with zero crossflow. To find such steady-state solutions, we obtain the following equation from (1.11) after making the substitution $\xi = \sqrt{3\lambda_*/2\alpha}\cdot x$:

$$h''' - \mathrm{sgn}(T_{10} - T_0)\left[\frac{1}{h(1 - h + \lambda_* h)^2} - G\right] h' = 0, \tag{1.12}$$

in which

$$G = -\frac{2\gamma \mathrm{sgn}(T_{10} - T_0)}{3\lambda_*} = \mp \frac{2\mathrm{sgn}(T_{10} - T_0)\rho g d^2}{3\lambda_* \sigma_T |T_{10} - T_0|}, \tag{1.13}$$

and the prime denotes differentiation with respect to ξ.

2. STABILITY OF EQUILIBRIUM OF A PLANE LAYER

The stability of the equilibrium state against long-wavelength perturbations can be analyzed on the basis of the long-wavelength convection model (1.9)–(1.11). Equation (1.11) has the solution $h \equiv h_0$, $h_0 \in (1, 0)$. It is evident from Eqs. (1.9) and (1.10) that this solution corresponds to the mechanical equilibrium state of a layer with a piecewise-linear temperature distribution along z. Linearizing Eq. (1.11) in the neighborhood of the equilibrium position $h = h_0$ and substituting the perturbed thickness $h = h_0 + \delta_{\exp}(\lambda_\omega t + i\omega x)$ (δ is a small amplitude) into the result, we obtain an equation for the growth rate λ_ω:

$$\lambda_\omega = -\frac{3\lambda_* \mathrm{sgn}(T_{10} - T_0) h_0^3}{2}(G_* - G)\omega^2 - \alpha\omega^4, \tag{2.1}$$

where the parameter G is given by Eq. (1.13), and

$$G_* = \frac{1}{h_0(1 - h_0 + \lambda_* h_0)^2}. \tag{2.2}$$

The last term in Eq. (2.1) can be disregarded in the limit $\omega \to 0$. The evolution of the long-wavelength perturbations obeys the product law $\mathrm{sgn}(T_{10} - T_0)(G_* - G)$.

The growth rate λ_ω is positive for a ceiling layer heated from the ceiling side, $T_0 > T_{10}$, where the parameter $G < 0$. A threshold of stability exists in the range of long-wavelength perturbations when the layer is heated at the free surface, i.e., when $T_0 < T_{10}$. The stability condition ($\lambda_\omega < 0$) has the form $G < G_*$. The equilibrium state of the ceiling layer heated on the side of the gaseous phase is unstable for $G > G_*$.

For a floor layer heated on the side of the gaseous phase, $T_{10} > T_0$, we have $G > 0$ and $\lambda_\omega < 0$. The equilibrium state is stable against long-wavelength perturbations. If it is heated from the floor side, $T_{10} < T_0$, the equilibrium is stable if $G > G_*$ and is unstable if $G < G_*$.

In these cases the condition for loss of stability of the equilibrium state of the layer against perturbations of infinite wavelength reduces to the equation $G = G_*$, in which G and G_* are expressed by Eqs. (1.13) and (2.2). We note that the equation $G = G_*$ can be obtained from the general equation describing the critical condition for the loss of stability of the equilibrium state of a heated two-layer fluid contained between parallel plates [1] by passing to the limit $\omega \to 0$, $\rho_* \to 0$, $\mu_* \to 0$, where ρ_* and μ_* are the density and dynamic viscosity ratios of the fluids.

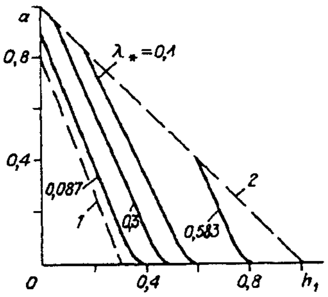

Fig. 1

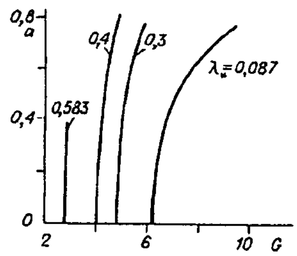

Fig. 2

3. STANDING WAVE WITH A MONOTONICALLY VARYING FREE SURFACE LEVEL

Equation (1.12) is invariant under the transformation $\xi \to \xi + \text{const}$. Let us assume for definiteness that the value of h for $\xi = 0$ is the same as the unperturbed value, i.e., $h(0) = h_0$. We investigate the solutions of Eq. (1.12) for which $h(\xi)$ is a monotonic function on $(-\infty, \infty)$ satisfying the conditions $h'(-\infty) = h'(\infty) = 0$. Since the function $h(\xi)$ is monotonic, we can regard $h'(\xi)$ as a function of h. Denoting by h_1 and h_2 ($h_1 < h_2$) the values of h obtained for $\xi = \mp\infty$ in the course of solving the equation, we obtain

$$h'|_{h=h_1} = h'|_{h=h_2} = 0. \tag{3.1}$$

Making use of Eq. (3.1) and integrating twice, from Eq. (1.12) we obtain the expression

$$h'^2 = (h_2 - h_1)^{-1}(h_2 - h)(h - h_1)F(h), \tag{3.2}$$

in which F(h) denotes the function

$$F = (\varphi - \varphi_1)(h - h_1)^{-1} + (\varphi - \varphi_2)(h_2 - h)^{-1}, \tag{3.3}$$

where

$$\varphi = \operatorname{sgn}(T_{10} - T_0)\,[2h\ln[h/(1 - h + \lambda_* h)] - Gh^2], \tag{3.4}$$

and φ_1 and φ_2 are the values of φ at the points h_1 and h_2.

Taking into account the invariance of Eq. (1.12) under the transformation $\xi \to -\xi$, we analyze the solutions for which $h(\xi)$ is a monotonically increasing function. From Eq. (3.2) we obtain the inverse dependence of ξ on h in the quadrature form

$$\xi = (h_2 - h_1)\int_{h_0}^{h}\frac{d\tau}{\sqrt{(h_2 - \tau)(\tau - h_1)F(\tau)}}. \tag{3.5}$$

The following condition must be satisfied in order for Eqs. (3.3)–(3.5) to describe a standing wave of the level elevation:

$$F(\tau) > 0,\ \tau \in (h_1, h_2),\ 0 < h_1 < h_2 < 1. \tag{3.6}$$

The quantity ξ must be equal to $-\infty$, $+\infty$ for $h = h_1, h_2$. From this result and from the validity of the expansions

$$F(\tau) \sim \varphi'(h_1) + (\varphi_1 - \varphi_2)/(h_2 - h_1) + o(\tau - h_1),$$

in the limit $\tau \to h_1$ and

$$F(\tau) \sim \varphi'(h_2) + (\varphi_1 - \varphi_2)/(h_2 - h_1) + o(\tau - h_2)$$

in the limit $\tau \to h_2$ we deduce the necessity of two more conditions

$$\varphi'(h_1) + (\varphi_1 - \varphi_2)(h_2 - h_1)^{-1} = 0; \tag{3.7}$$

$$\varphi'(h_2) + (\varphi_1 - \varphi_2)(h_2 - h_1)^{-1} = 0. \tag{3.8}$$

It is evident from Eqs. (3.3)-(3.5) that the profile of the free surface depends on the four dimensionless parameters λ_*, G, h_1, and h_2. By virtue of conditions (3.7) and (3.8) only two of these parameters are independent.

Substituting expression (3.4) and carrying out several transformations, from (3.7) and (3.8) we obtain the equations

$$\ln\frac{h_1(1 - h_2 + \lambda_* h_2)}{h_2(1 - h_1 + \lambda_* h_1)} + \frac{(h_2 - h_1)[2 - (1 - \lambda_*)h_1 - (1 - \lambda_*)h_2]}{(h_1 + h_2)(1 - h_1 + \lambda_* h_1)(1 - h_2 + \lambda_* h_2)} = 0; \tag{3.9}$$

$$G = \frac{2}{(1 - h_1 + \lambda_* h_1)(1 - h_2 + \lambda_* h_2)(h_1 + h_2)}, \tag{3.10}$$

which interrelate the parameters λ_*, G, h_1, and h_2.

It is evident from Eq. (2.10) that a solution can exist only for $G > 0$. For a floor layer ($\gamma > 0$), according to (1.13), a solution can exist only if the layer is heated from the floor side. For a ceiling layer ($\gamma < 0$), according to (1.13), a solution can exist for $T_{10} > T_0$, i.e., if the layer is heated from the side of the free surface. Figure 1 shows the standing wave amplitude $a = h_2 - h_1$ as a function of the minimum level h_1 for certain values of λ_*. This dependence is plotted from the results of the numerical solution of Eq. (3.9). The dashed lines 1 and 2 correspond to the limits $\lambda_* \to 0$ and $a \to 1 - h_1$ and define the boundary of the region in which solutions in the form of a standing wave of the level elevation can exist. The validity of inequality (3.6) in this region has been verified numerically. We find that this inequality is satisfied everywhere in the region for a ceiling layer but not for a floor layer. Consequently, solutions in the form of a standing wave of the level elevation exist only for a ceiling layer heated on the side of the free surface.

It is evident from Fig. 1 that the standing wave amplitude becomes a maximum either in the limit $h_1 \to 0$ ($\lambda_* < \lambda_*^0 \approx 0.22$) or in the limit $h_2 \to 1$ ($\lambda_* > \lambda_*^0$). For $\lambda_* > \lambda_*^0$ the maximum amplitude decreases with increasing value of λ_*, tending to zero as $\lambda_* \to 2/3$. For $a = 0$ we have $h_2 = h_1 = h_0$. Consequently, for a given value of λ_* the $a(h_1)$ curve describes the amplitudes of steady-state solutions corresponding to an equilibrium state with $h = h_0$. The velocity, pressure, and temperature for these solutions are found in terms of $h(\xi)$ according to Eqs. (1.9) and (1.10).

The value $\lambda_* = 0.087$ corresponds to the system glycerin–air at a temperature of 15°C, and $\lambda_* = 0.583$ corresponds to the system glycerin–olive oil at room temperature, where olive oil takes the role of the "gaseous phase." The ratio $\mu_* = \mu_1/\mu$ of the dynamic viscosity coefficient of olive oil μ_1 to that of glycerin μ is equal to 0.043. Consequently, the dynamical influence of a layer of olive oil on a layer of glycerin can be disregarded.

The thickness h_0 at which branching of the equilibrium state takes place depends on the parameter λ_*. The analytical dependence of h_0 on λ_* is obtained by representing Eq. (3.9) by a series expansion in the small amplitude in the neighborhood of the equilibrium state with $h_1 = h_2 = h_0$ and setting the first nonvanishing term of the expansion [which is $O(a^3)$] equal to zero. This dependence is given by the equation

$$h_0 = 1/(3(1 - \lambda_*)), \tag{3.11}$$

which is derived in [3], where an analytical representation is given for a small-amplitude solution. Thus, steady-state convection with a monotonically varying free surface level can exist in a ceiling liquid layer only in the special case when the thickness

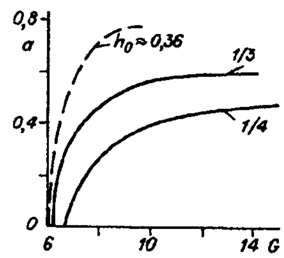

Fig. 3

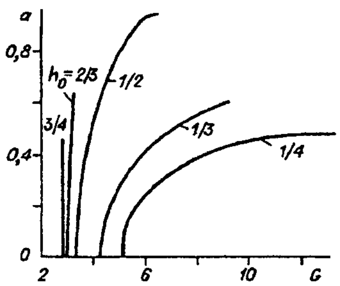

Fig. 4

h_0 of the layer in the equilibrium state is related to the ratio of the thermal conductivities of the gas and the liquid by Eq. (3.11).

Figure 2 shows the wave amplitude a as a function of the parameter G, plotted by means of Eqs. (3.9) and (3.10). The critical values G_* at which branched steady-state solutions appear are equal to 6.16, 4.73, 4.05, and 2.81 for $\lambda_* = 0.087$, 0.3, 0.4, and 0.583, respectively. All branched solutions exist in the supercritical region $G > G_*$. It is reasonable to expect that they will be stable. Clearly, the interval of values of the parameter G wherein branched solutions exist decreases as λ_* increases.

For $\lambda_* = 0.583$ the parameter G varies from $G_* = 2.81$ to the value $G = 2.86$, at which the wave amplitude is equal to 0.4. For the system glycerin–olive oil the parameter $\sigma_T \approx 0.2$ dyn/(cm·°C), and the difference in the densities is $\rho = 0.342$ g/cm^3. If the acceleration of gravity is equal to $10^{-2}g_0$, we infer from Eq. (1.13) that $G_* = 2.81$ is attained at the critical differential temperature $(T_{10} - T_0) = 6.82$°C, and $G = 2.86$ is attained for $T_{10} - T_0 = 6.69$°C. Consequently, when the differential temperature across the plates decreases by 0.13°C, the standing wave amplitude increases from zero to 0.4, at which $h_2 = 1$ (a ceiling layer of glycerin comes into contact with the opposite plate at infinity).

For an acceleration of gravity $g = 10^{-2}g_0$ and $d = 1$ cm, estimates based on Eq. (1.13) for a ceiling layer of glycerin show that branched steady-state solutions occur at a differential temperature $(T_{10} - T_0)_* \approx 37.5$°C and exit up to $T_{10} - T_0 = 18.2$°C.

4. STANDING WAVES WITH A CONVEXITY TOWARD THE GASEOUS PHASE

We now investigate the existence of solutions of Eq. (3.2) in the form of a standing solitary wave with a convexity facing the gaseous phase in the neighborhood of $\xi = 0$. We have $h = h_1$ for $\xi = \pm\infty$ and $h = h_2$ for $\xi = 0$, where $h_1 < h_2$. The solution must satisfy conditions (3.1) and be described in the interval $(-\infty, 0)$ by Eq. (3.5) with h_2 as the lower limit of integration. The value of ξ in Eq. (3.5) must tend to $-\infty$ in the limit $h \to h_1$. Now only the one equation (3.7) has to be satisfied instead of both Eq. (3.7) and Eq. (3.8). From Eq. (3.7), taking Eq. (3.4) into account, we obtain the dependence of the parameter G on the levels h_1 and h_2:

$$G = \frac{2}{(h_2 - h_1)^2}\left[h_2 \ln\frac{h_2(1 - h_1 + \lambda_* h_1)}{h_1(1 - h_2 + \lambda_* h_2)} - \frac{h_2 - h_1}{1 - h_1 + \lambda_* h_1}\right]. \tag{4.1}$$

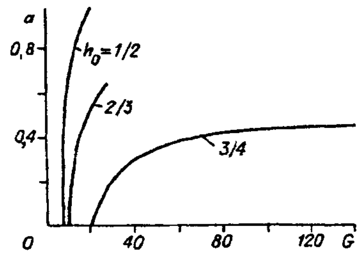

Fig. 5

In general, the profile of the free surface depends on three parameters: h_1, h_2, and λ_*. Passing to the limit $h_2 \to h_1 = h_0$ in Eq. (4.1), we infer that $G \to G_*$, where G_* is given by Eq. (2.2). In the calculations we analyze the two-parameter family of solutions for which the average level $(h_1 + h_2)/2$ is constant and equal to the unperturbed level h_0. The value of λ_* is varied from 0 to 1.5. For $\lambda_* \in (0, 1.5)$ and $h_0 \in (0, 1)$ the value of G given by Eq. (4.1) is positive, so that, according to (1.13), a solution can exist for a ceiling layer if $T_{10} > T_0$ and for a floor layer if $T_{10} < T_0$. The sign of F depends on $\mathrm{sgn}(T_{10} - T_0)$. A positive value of F(h) for $h \in (h_1, h_2)$ implies the existence of a standing wave with a convexity toward the gaseous phase for a ceiling layer, and a negative value implies the same for a floor layer. If F is not an alternating function for $h \in [h_1, h_2]$, solutions of the type in question do not exist for either a floor layer or a ceiling layer.

An analysis of the sign of the function F(h) indicates the following. For a ceiling liquid layer heated on the side of the gaseous phase a steady-state solution in the form of a standing wave with a convexity toward the gaseous phase exists for any $\lambda_* \in (0, 1.5]$ if the thickness of the undisturbed layer $h_0 \leq 1/3$. Such a solution exists for $h_0 > 1/3$ if the thermal conductivity ratio λ_* lies in the interval $[\lambda_*(h_0), 1.5]$, where

$$\lambda_*(h_0) = (3h_0 - 1)/3h_0. \tag{4.2}$$

If $h_0 > 1/3$ and $\lambda_* = \lambda_*(h_0)$, then the function $F(h) \to 0$ in the limit $h \to h_2$, and Eq. (3.8) is satisfied. The integral in Eq. (3.5) diverges as $h \to h_2$. The solution degenerates into a solution of the level-elevation standing wave type. If $h_0 > 1/3$ and $0 < \lambda_* < \lambda_*(h_0)$, solutions of the standing wave type with a convexity do not exist for either a floor layer or a ceiling layer. A solution is also nonexistent for a floor layer if $h_0 < 1/3$.

The branching pattern of the equilibrium state of a planar ceiling layer of glycerin is shown in Fig. 3 for the case when the lower medium is air ($\lambda_* = 0.082$) and in Fig. 4 for the case when the role of the "gaseous phase" is taken by olive oil ($\lambda_* = 0.503$). Figure 3 shows the wave amplitude a as a function of the parameter G for $h_0 = 1/4$ and $h_0 = 1/3$. The dashed line corresponds to the value $h_0 = 1/[3(1 - \lambda_*)] \approx 0.36$, at which the solution degenerates into a standing wave of the level elevation. The branches in Fig. 4 correspond to $h_0 = 1/4$, 1/3, 1/2, 2/3, and 3/4. The solution degenerates for $h_0 = 1/[3(1 - \lambda_*)] \approx 0.84$.

The value $a = 0$ is stable for the undisturbed layer when the parameter G is smaller than the value given by Eq. (2.2): $G_*(h_0, \lambda_*)$, at which the solution branches. All branched solutions exist in the supercritical region.

5. STANDING THERMOCAPILLARY WAVES

Equations (3.3)–(3.5) describe a solution in the form of a standing solitary wave with a convexity facing the gaseous phase in the neighborhood of $\xi = 0$ if the lower limit of integration in Eq. (3.5) is $h = h_1$ and the integrand has a nonintegrable singularity for $h = h_2$ ($h_2 > h_1$). From Eqs. (3.8) and (3.4) we find the dependence of the parameter G on h_1, h_2, and λ_*:

$$G = \frac{2}{(h_2 - h_1)^2}\left[h_1 \ln\frac{h_1(1 - h_2 + \lambda_* h_2)}{h_2(1 - h_1 + \lambda h_1)} + \frac{h_2 - h_1}{1 - h_2 + \lambda_* h_2}\right]. \tag{5.1}$$

In the limit $h_2 \to h_1 = h_0$ we infer from Eq. (5.1) that the parameter G tends to the value G_* given by Eq. (2.2). We have investigated solutions for which $(h_1 + h_2)/2 = h_0$ in the case $\lambda_* \in (0, 1.5)$. The analysis of the wave amplitude $a = h_2 - h_1$ as a function of the parameters G_* and λ_* and the sign of the function F in the interval (h_1, h_2) yields the following result.

For a ceiling layer heated on the side of the gaseous phase a steady-state solution in the form of a standing wave with a convexity toward the gaseous phase exists if the thickness of the undisturbed layer $h_0 \geq 1/3$ and the parameter λ_* belongs to the interval $(0, \lambda_*(h_0))$, where $\lambda_*(h_0)$ is given by Eq. (4.2). Branching takes place in the supercritical region. For $h_0 < 1/3$ a solution of the investigated type does not exist for a ceiling liquid layer. In the case of a floor layer a solution with a concavity toward the gaseous phase does not exist for any value of $h_0 \in (0, 1)$.

Figure 5 shows the amplitudes of the branched solutions as functions of the parameter G for three values of the thickness h_0 in the system glycerin–air (with the glycerin layer on the ceiling).

For all values of h_0 the solutions cease to exist when the thickness $h_2 \to 1$ for $\xi = \pm\infty$. Here the minimum level $h_1 \to 0$ for $h_0 = 1/2$. In the limit $h_2 \to 1$ we have $h_1 \to 0.5$ if $h_0 = 3/4$, and we have $h_1 \to 1/3$ if $h_0 = 2/3$. Estimates show that branched solutions emerge for an acceleration of gravity $g = 10^{-3}g_0$ when the critical differential temperature is equal to 23.65°C, 47.31°C, and 70.01°C for thicknesses $h_0 = 3/4, 2/3$, and 1/2, respectively. The differential temperature decreases along each branch. For $h_2 = 0.98$ it is equal to 3.49°C, 18.82°C, and 27.6°C for $h_1 = 3/4, 2/3$, and 1/2.

The function $h(\xi)$ is even for a standing wave convex or concave toward the gaseous phase. For $\xi = 0$ we have $h' = h''' = 0$ and, according to Eqs. (1.9) and (1.10), the longitudinal velocity component $u = 0$. The flow therefore has a double-vortex structure.

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